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PREHEATING AND REHEATING IN INFLATIONARY COSMOLOGY: A PEDAGOGICAL SURVEY

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Recent progress in the preheating phenomena for inflationary cosmology is reviewed. We first discuss estimates of the preheating time scale and particle production at the early stages of parametric amplification within the Mathieu and Lamé approximations and we analyze their precision and limitations. The necessity of self-consistent calculations including the non-linearity of the field theory equations in an energy conserving scheme is stressed. The large N calculations including the field back-reaction are reviewed. For spontaneously broken theories the issue of symmetry restoration is analyzed. A discussion of the possibility and criterion for symmetry restoration is presented. (To appear in the Proceedings of the Paris Euronetwork Meeting 'String Gravity', Observatoire de Paris, June 1996).

I. INTRODUCTION

Research activity on inflationary cosmologies has continued steadily since the concept of inflationary cosmology was first proposed in 1981 [1].

It was recognized that in order to merge an inflationary scenario with standard Big Bang cosmology a mechanism to reheat the universe was needed. Such a mechanism must be present in any inflationary model to raise the temperature of the Universe at the end of inflation, thus the problem of reheating acquired further importance deserving more careful investigation. The original version of reheating envisaged that during the last stages of inflation when the accelerated universe expansion slows down, the energy stored in the oscillations of the inflaton zero mode transforms into particles via single particle decay. Such particle production reheats the universe whose temperature reduced enormously due to the inflationary expansion [2].

It was realized recently [4,15,16,18], that in fact, the elementary theory of reheating [2] does not describe accurately the quantum dynamics of the fields.

Our programme on non-equilibrium dynamics of quantum field theory, started in 1992 [3], is naturally poised to provide a framework to study these problems. The larger goal of the program is to study the dynamics of non-equilibrium processes from a fundamental field-theoretical description, by solving the dynamical equations of motion of the underlying four dimensional quantum field theory for physically relevant problems: phase transitions out of equilibrium, particle production out of equilibrium, symmetry breaking and dissipative processes.

The focus of our work is to describe the quantum field dynamics when the energy density is **high**. That is, a large number of particles per volume m^{-3}, where m is the typical mass scale in the theory. Usual S-matrix calculations apply in the opposite limit of low energy density and since they only provide information on *in* $\rightarrow$ *out* matrix elements, are unsuitable for calculations of expectation values. Our methods were naturally applied to different physical problems like pion condensates [5,7], supercooled phase transitions [3,6] and inflationary cosmology [4,6,8–10].

An analogous program has been pursued by the Los Alamos group, whose research focuses on non-linear effects in scalar QED linked to the pair production in strong electric fields [11], the Schwinger-Keldysh non-equilibrium formalism in the large N expansion [12], and the dynamics of chiral condensates in such framework [13].

II. PREHEATING IN INFLATIONARY UNIVERSES

As usual in inflationary cosmology, matter is described in an effective way by a self-coupled scalar field $\Phi(x)$ called the inflaton. The spacetime geometry is a cosmological spacetime with metric $ds^2 = (dt)^2 - a(t)^2 \; (d\vec{x})^2$, where $a(t)$ is the scale factor.

The evolution equations for the k-modes of the inflaton field as considered by different groups can be summarized as follows,

$$\ddot{\chi}_k + 3H(t)\;\dot{\chi}_k + \left(\frac{k^2}{a^2(t)} + M^2[\Phi(.)]\right)\chi_k(t) = 0 \quad (2.1)$$

where $H(t) = \dot{a}(t)/a(t)$, and $M^2[\Phi(.)]$ is the effective mass felt by the modes. The expression considered depends on the model (here the $\lambda\ \Phi^4$ model) and the approximations made. The value of λ is scenario-dependent but it is usually very small. $M^2[\Phi(.)]$ depends on the scale factor and on the physical state. Therefore, it depends on the modes $\chi_k(t)$ themselves in a complicated way. One is definitely faced with a complicated non-linear problem. We call 'back-reaction' the effect of the modes $\chi_k(t)$ back on themselves through $M^2[\Phi(.)]$.

In the initial stage, all the energy is assumed in the zero mode of the field $\phi(t)$ [3,6–8,15–18]. That is, the field expectation value $\phi(t) \equiv < \Phi(x) >$, where $< \cdots >$ stays for the expectation value in the translational invariant but non-equilibrium quantum state. For very weakly coupled theories, and early times, such that the back-reaction effects of the non-equilibrium quantum fluctuations can be neglected, one can approximate,

$$M^2[\Phi(.)] \simeq m^2 + \frac{\lambda}{2}\phi(t)^2 \tag{2.2}$$

At this moment, the scale factor is set to be a constant in ref. [15,18]. Refs. [3,6–8] consider Minkowski spacetime from the start. In ref. [16], the scale factor is set to be a constant for a model without $\lambda\Phi^4$ inflaton self-coupling. That is, for the classical potential [16]

$$V = \frac{1}{2}m^2\Phi^2 + g\sigma^2\Phi^2 \ , \tag{2.3}$$

one can consider as classical solution $\Phi(t) = \Phi_0\ \cos(mt)$, $\sigma = 0$. In such case, the Mathieu equation approximation is exact in Minkowski spacetime. However, the potential (2.3) is unstable under renormalization (a Φ^4 counterterm is needed from the one-loop level). Hence, the $\lambda = 0$ choice is a fine-tuning not protected by any symmetry. In a second case a massless selfcoupled $\lambda\Phi^4$ field in a radiation dominated universe $a(t) \propto \sqrt{t}$ is considered in conformal time [16]. In such specific case the classical field equations take the Minkowski form for $\sqrt{t}\Phi$.

In a way or another, using the classical oscillating behaviour of $\phi(t)$, one is led by eq.(2.1) to an effective mass that oscillates in time. In this approximation (which indeed, may be very good for small coupling, see [8]), eq.(2.1) exhibits **parametric resonance** as noticed first in ref [15]. Namely, there are allowed and forbidden bands in k^2. The modes within the forbidden bands grow exponentially whereas those in the allowed bands stay with bounded modulus. The growth of the modes in the forbidden bands translates into profuse particle production, the particles being created with these particular unstable momenta. The rate of particle production is determined by the imaginary part of the Floquet index in these unstable bands. Notice that the approximation (2.2) breaks down as soon as many particles are produced. Namely, when the energy of the produced particles becomes of the order of the zero-mode energy and eq.(2.2) is no more valid.

Now, in order to compute quantitatively the particles produced one needs the form of $\phi(t)$. In ref. [15,16,18] $\phi(t)$ is approximated by a cosinus function in the calculations. The mode equations become a Mathieu equation (the scale factor is set to be a constant). In ref. [15] the Bogoliubov-Krylov approximation is used to compute estimates. In ref. [16,17], estimates are obtained using asymptotic formulas for the Mathieu equation. In ref. [8] the exact classical solution is used (a cn Jacobi function) to compute estimates.

Let us now compare the results from the exact mode solutions obtained in ref. [8] with the Mathieu equation approximation to it. In units where $m^2 = 1$ and setting $\eta(t) \equiv \sqrt{\frac{\lambda}{2}}\ \phi(t)$, one finds

$$\eta(t) = \eta_0\ \mathrm{cn}\left(t\sqrt{1+\eta_0^2}, \bar{k}\right)$$
$$\bar{k} = \frac{\eta_0}{\sqrt{2(1+\eta_0^2)}}\ , \tag{2.4}$$

where cn stands for the Jacobi cosinus and we choose for initial conditions $\eta(0) = \eta_0$, $\dot{\eta}(0) = 0$.

Here $\eta(t)$ has period $4\omega \equiv 4\,K(\bar{k})/\sqrt{1+\eta_0^2}$, where $K(\bar{k})$ is the complete elliptic integral of first kind, and $\eta(t)^2$ has period 2ω.

Inserting this form for $\eta(\tau)$ in eqs.(2.2) and (2.1) yields

$$\left[\frac{d^2}{dt^2} + k^2 + 1 + \eta_0^2\ \mathrm{cn}^2\left(t\sqrt{1+\eta_0^2}, \bar{k}\right)\right]\chi_k(t) = 0\ . \tag{2.5}$$

This is the Lamé equation for a particular value of the coefficients that make it solvable in terms of Jacobi functions [24]. As shown in ref. [8], this equation has only one forbidden band for positive k^2 going from $k^2 = 0$ to $k^2 = \frac{\eta_0^2}{2}$. One can choose Floquet solutions of eq.(2.5) that fullfil the relation

$$U_k(t + 2\omega) = e^{iF(k)}\ U_k(t), \tag{2.6}$$

where the Floquet indices $F(k)$ are independent of t. In the forbidden band the $F(k)$ posses an imaginary part. The production rate is determined by the imaginary part of the Floquet index.

The exact form of $F(k)$ results [8],

$$F(k) = -2iK(\bar{k})\ Z(2K(\bar{k})\,v) + \pi$$

where $Z(u)$ is the Jacobi zeta function [25] and v is a function of k in the forbidden band defined by

$$k = \frac{\eta_0}{\sqrt{2}}\,\mathrm{cn}(2K(\bar{k})\,v, k)\ ,\ 0 \leq v \leq \frac{1}{2}. \tag{2.7}$$

All these elliptic functions posses fastly convergent expansions in powers of the elliptic nome

$$q \equiv e^{-\pi K'(\bar{k})/K(\bar{k})}\ .$$

Since $0 \leq \bar{k} \leq 1/\sqrt{2}$ [see eq.(2.4)], we have

$$0 \leq q \leq e^{-\pi} = 0.0432139\ldots \ . \tag{2.8}$$

Then,

$$F(k) = 4i\,\pi\, q\, \sin(2\pi v)\ \left[1 + 2\,q\, \cos 2\pi v + O(q^2)\right] + \pi\ . \tag{2.9}$$

The imaginary part of this function has a maximum at $k = k_1 = \frac{1}{2}\,\eta_0\,(1-q) + O(q^2)$ where [8]

$$\mathcal{F} \equiv Im F(k_1) = 4\,\pi\, q + O(q^3)\ . \tag{2.10}$$

This simple formula gives the maximum of the imaginary part of the Floquet index in the forbidden band with a precision better than $8.\,10^{-5}$. q can be expressed in terms of η_0 as follows [8]

$$q = \frac{1}{2}\,\frac{(1+\eta_0^2)^{1/4} - (1+\eta_0^2/2)^{1/4}}{(1+\eta_0^2)^{1/4} + (1+\eta_0^2/2)^{1/4}}\ .$$

with an error smaller than $\sim 10^{-7}$.

Let us now proceed to the Mathieu equation analysis of this problem. The cn Jacobi function can be expanded as [26]

$$\mathrm{cn}(z,\bar{k}) = (1-q)\cos(1-4q)z + q\cos 3z + O(q^2)\ .$$

To zeroth order in q we have

$$\eta(t)^2 = \frac{\eta_0^2}{2}\left[1 + \cos(2t\sqrt{1+\eta_0^2})\right] + O(q)\ .$$

and $2\omega = \pi/\sqrt{1+\eta_0^2} + O(q)$. Under such approximations eq.(2.5) becomes the Mathieu equation [27]

$$\frac{d^2y}{dz^2} + (a - 2\bar{q}\cos 2z)\,y(z) = 0\ , \tag{2.11}$$

where

$$a = 1 + \frac{k^2 - \frac{\eta_0^2}{2}}{\eta_0^2 + 1}\ ,\ \bar{q} = \frac{\eta_0^2}{4(\eta_0^2+1)}$$

and $z = \sqrt{\eta_0^2+1}\ t$. Notice that $0 \leq \bar{q} \leq 1/4$ in the present case. Eq.(2.11) posses an infinite number of forbidden bands for $k^2 > 0$. The lower and upper edge of the first band are respectively [27]

$$k_{inf}^2 = \frac{\eta_0^2}{4}\left[1 - \frac{\eta_0^2}{2^5(\eta_0^2+1)} + \frac{\eta_0^4}{2^{10}(\eta_0^2+1)} + \ldots\right]\ ,$$

till

$$k_{sup}^2 = \frac{\eta_0^2}{4}\left[3 - \frac{\eta_0^2}{2^5(\eta_0^2+1)} - \frac{\eta_0^4}{2^{10}(\eta_0^2+1)} + \ldots\right]\ .$$

These values must be compared with the exact result for the Lamé equation (2.5) : $k_{inf}^2 = 0\ ,\ k_{sup}^2 = \frac{\eta_0^2}{2}$. The width of the band is well approximated but not its absolute position. The numerical values of the maximum of the imaginary part of the Floquet index are given in Table I and compared with the exact values from eq.(2.10).

We see that the Mathieu approximation **underestimates** the exact result by a fraction ranging from 13% to 39%. The second forbidden band in the Mathieu equation yields $\mathcal{F}_{Mathieu} = 0.086\ldots$ for $\eta_0 \to \infty$. This must be compared with $\mathcal{F}_{Lame} = 0$ (no further forbidden bands).

In ref. [17], an even larger discrepancy between Lame and Mathieu Floquet indices has been reported within a different approximation scheme.

It must be noticed that the Floquet indices entering in exponents produces a dangerous error propagation. For example, the number of particles produced during reheating is of the order of the exponential of $2\mathcal{F}$ times the reheating time in units of $\pi/\sqrt{1+\eta_0^2}$. An error of 25% in $\mathcal{F}$ means an error of 25% in the exponent. (For instance, one would find 10^9 instead of 10^{12}).

The mode equations (2.1) apply to the self-coupled $\lambda\ \Phi^4$ scalar field. Models for reheating usually contain at least two fields: the inflaton and a lighter field $\sigma(x)$ in which the inflaton decays. For a $g\sigma^2\Phi^2$ coupling, the mode equations for the σ field take the form [7,16,18]

$$\ddot{V}_k + 3H(t)\dot{V}_k + \left(\frac{k^2}{a^2(t)} + m_\sigma^2 + \frac{g}{\lambda}F[\Phi(.),\sigma(.)]\right)V_k(t) = 0 \tag{2.12}$$

A new dimensionless parameter $\frac{g}{\lambda}$ appears here. Neglecting the σ and Φ backreaction, we have

$$F[\Phi(.),\sigma(.)] \simeq \eta^2(t)\ . \tag{2.13}$$

In ref. [6,8], it is shown that abundant particle production (appropriate for reheating) shows up even for $g = \lambda$.

η_0	$\mathcal{F}_{Lame}$	$\mathcal{F}_{Mathieu}$	$\%error$
1	$0.2258\ldots$	$0.20\ldots$	13%
4	$0.4985\ldots$	$0.37\ldots$	35%
$\eta_0 \to \infty$	$4\pi e^{-\pi} = 0.5430\ldots$	$0.39\ldots$	39%

TABLE I. The maximum of the imaginary part of the Floquet index $\mathcal{F}$ for the Lamé equation and for its Mathieu approximation.

Eqs.(2.12)-(2.13) becomes a Lamé equation when $\eta(t)$ is approximated by the classical solution in Minkowski spacetime given by (2.4). Such Lamé equation is solvable in closed form when the couplings g and λ are related as follows [8]

$$\frac{2g}{\lambda} = n(n+1)\ ,\ n = 1, 2, 3, \ldots$$

In those cases there are n forbidden bands for $k^2 \geq 0$. The Lamé equation exhibits an infinite number of forbidden bands for generic values of $\frac{g}{\lambda}$. The Mathieu and WKB approximations has been applied in the non-exactly solvable cases [16–18]. However, as the above analysis shows (see Table I) such estimations cannot be trusted quantitatively. The only available precise method consists on accurate numerical calculations as those of ref. [6–8] (where the precision is at least 10^{-6}).

Estimates in the cosinus approximation for FRW-de Sitter backgrounds and open universes using the Bogoliubov-Krylov approximation are given in ref. [21]. In ref. [22] the Bogoliubov-Krylov approximation is applied to the large N equations.

Applications of preheating to various relevant aspects of the early cosmology are considered in ref. [23].

As soon as the quantum fluctuations grow and cease to be negligible compared with the the classical piece (2.13), all the approximations discussed so far (Lamé, Mathieu, etc.) break down. This time is the so-called preheating time t_{reh} [8]. One can estimate t_{reh} by equating the zero mode energy (2.13) with the estimation of the quantum fluctuations derived from the unstable Floquet modes [16,8]. Such estimation yields when the Lamé Floquet indices are used [8],

$$t_{reh} \approx \frac{1}{B}\ \log \frac{N(1+\eta_0^2/2)}{g\sqrt{B}}\ , \tag{2.14}$$

where

$$B = 8\,\sqrt{1+\eta_0^2}\ q\ (1-4q) + O(q^3)\ ,$$
$$N = \frac{4}{\sqrt{\pi}}\ \sqrt{q}\ \frac{(4+3\,\eta_0^2)\,\sqrt{4+5\,\eta_0^2}}{\eta_0^3\,(1+\eta_0^2)^{3/4}}\,[1+O(q)] \quad . \tag{2.15}$$

Where B is determined by the maximum of the imaginary part of the Floquet index in the band. It is now clear that a few percent correction to the Floquet indices will result in a large error in the estimate of the preheating time scale.

III. THE END OF PREHEATING AND BEYOND: SELF-CONSISTENT LARGE N CALCULATIONS

In order to compute physical magnitudes beyond t_{reh}, one **must** solve self-consistently the field equations including the back reaction. In ref. [6,8] this is done for the $N \to \infty$ limit and in ref. [7] to one-loop order.

In the large N limit the zero mode and k-modes renormalized equations take the form,

$$\ddot{\eta} + \eta + \eta^3 + g\ \eta(t)\,\Sigma(t) = 0\ , \tag{3.1}$$
$$\left[\ \frac{d^2}{dt^2} + k^2 + 1 +\ \eta(t)^2 + g\ \Sigma(t)\ \right]\varphi_k(t) = 0\ , \tag{3.2}$$

where $g\Sigma(t)$ is given by

$$g\Sigma(t) = g\int_0^\infty k^2 dk\,\left\{\mid \varphi_k(t)\mid^2 - \frac{1}{\Omega_k}\right.$$
$$\left. + \frac{\theta(k-\kappa)}{2k^3}\left[-\eta_0^2 + \eta^2(t) + g\ \Sigma(t)\right]\right\}\ ,$$
$$\Omega_k = \sqrt{k^2+1+\eta_0^2}\ . \tag{3.3}$$

We choose the initial state such that at $t=0$ the quantum fluctuations are in the ground state of the oscillators. That is,

$$\varphi_k(0) = \frac{1}{\sqrt{\Omega_k}} \quad , \quad \dot{\varphi}_k(0) = -i\ \sqrt{\Omega_k}\ ,$$

$$\eta(0) = \eta_0 \quad , \quad \dot{\eta}(0) = 0\ .$$

In the one-loop approximation the term $g\ \Sigma(t)$ is absent in eq.(3.2).

Eqs.(3.2)-(3.3) were generalized to cosmological spacetimes (including the renormalization aspects) in ref. [4].

Eqs.(3.2) is an infinite set of coupled non-linear differential equations in the k-modes $\varphi_k(t)$ and in the zero mode $\eta(t)$. We have numerically solved eqs.(3.2) for a variety of couplings and initial conditions [6–8]. In figs. 1-3 we display $\eta(t)$, $\Sigma(t)$ and the total number of created particles $N(t)$ as a function of t for $\eta_0 = 4$ and $g = 10^{-12}$ [8]. One sees that the quantum fluctuations are indeed very small till we approach the reheating time t_{reh}. As one can see from the figures, eq.(2.14) gives a very good approximation for t_{reh} and for the behaviour of the quantum fluctuations for $t \leq t_{reh}$. For times earlier than t_{reh}, the Lamé estimate for the envelope of $\Sigma(t)$ and $N(t)$ take the form [8],

$$\Sigma_{est-env}(t) = \frac{1}{N\,\sqrt{t}}\ e^{B\,t}\ ,$$
$$N_{est-env}(t) = \frac{1}{8\pi^2}\ \frac{4+\frac{9}{2}\eta_0^2}{\sqrt{4+5\eta_0^2}}\ \Sigma_{est-env}(t)\ . \tag{3.4}$$

where B and N are given by eq.(2.15).

This analysis presents a very clear physical picture of the properties of the "gas" of particles created during the preheating stage. This "gas" turns to be **out** of thermodynamic equilibrium as shown by the energy spectrum obtained, but it still fulfills a radiation-like equation of state $p \simeq e/3$ [8].

In approximations that do not include the backreaction effects there is infinite particle production, since these

do not maintain energy conservation [15,18]. The full backreaction problem and the approximation used in our work exactly conserves the energy. As a result, particle production shuts-off when the back-reaction becomes important ending the preheating stage.

In ref. [14] results of ref. [6] are rederived with a different renormalization scheme.

IV. BROKEN SYMMETRY AND ITS QUANTUM RESTORATION

Up to now we discussed the unbroken symmetry case $m^2 = 1 > 0$. In the spontaneously broken symmetry case $m^2 = -1 < 0$, the classical potential takes the form

$$V(\eta) = \frac{1}{4}(\eta^2 - 1)^2 \tag{4.1}$$

In refs. [6,8] the field evolution is numerically solved in the large N limit for $m^2 < 0$. Analytic estimations are given in ref. [8] for early times.

In ref. [6] it is shown that for small couplings and small η_0, the order parameter damps very quickly to a constant value close to the origin. That is, the order parameter ends close to the classical false vacuum and far from the classical minimum $\eta = 1$. The fast damping is explained, in this spontaneously broken symmetry case by the presence of Goldstone bosons. The massless Goldstone bosons dissipate very efficiently the energy from the zero mode. We plot in figs. 4 and 5, $\eta(t)$ and $\Sigma(t)$ for $\eta_0 = 10^{-5}$ and $g = 10^{-12}$ [8]. The profuse particle production here is due to spinodal unstabilities.

In ref. [19], it is claimed that quantum fluctuations have restored the symmetry in the situations considered in ref. [6]. However, the presence of massless Goldstone bosons and the non-zero limiting value of the order parameter clearly show that the symmetry **is** always broken in the cases considered in ref. [6,8]. These results rule out the claim in ref. [19]. In particular, the asymptotic value of the order parameter results from a very detailed dynamical evolution that conserves energy.

In the situation of 'chaotic initial conditions' but with a broken symmetry tree level potential, the issue of symmetry breaking is more subtle. In this case the zero mode is initially displaced with a large amplitude and very high in the potential hill. The total energy *density* is non-perturbatively large. Classically the zero mode will undergo oscillatory behavior between the two classical turning points, of very large amplitude and the dynamics will probe both broken symmetry states. Even at the classical level the symmetry is respected by the dynamics in the sense that the time evolution of the zero mode samples equally both vacua. This is not the situation that is envisaged in usual symmetry breaking scenarios. For broken symmetry situations there are no finite energy field configurations that can sample both vacua. In the case under consideration with the zero mode of the scalar field with very large amplitude and with an energy density much larger than the top of the potential hill, there is enough energy in the system to sample both vacua. (The energy is proportional to the spatial volume). Parametric amplification transfers energy from the zero mode to the quantum fluctuations. Even when only a fraction of the energy of the zero mode is transferred thus creating a non-perturbatively large number of particles, the energy in the fluctuations is very large, and the equal time two-point correlation function is non-perturbatively large and the field fluctuations are large enough to sample both vacua. The evolution of the zero mode is damped because of this transfer of energy, but in most generic situations it does not reach an asymptotic time-independent value, but oscillates around zero, sampling the tree level minima with equal probability. This situation is reminiscent of finite temperature in which case the energy density is finite and above a critical temperature the ensemble averages sample both tree level vacua with equal probability thus restoring the symmetry. In the dynamical case, the "symmetry restoration" is just a consequence of the fact that there is a very large energy density in the initial state, much larger than the top of the tree level potential, thus under the dynamical evolution the system samples both vacua equally. This statement is simply the dynamical equivalent of the equilibrium finite temperature statement that the energy in the quantum fluctuations is large enough that the fluctuations can actually sample both vacua with equal probability.

Thus the criterion for symmetry restoration when the tree level potential allows for broken symmetry states is that the energy density in the initial state be larger than the top of the tree level potential. That is when the amplitude of the zero mode is such that $V(\eta_0) > V(0)$. In this case the dynamics will be very similar to the unbroken symmetry case, the amplitude of the zero mode will damp out, transferring energy to the quantum fluctuations via parametric amplification, but asymptotically oscillating around zero with a fairly large amplitude.

To illustrate this point clearly, we plot in figs. 6 and 7, $\eta(t)$ and $\Sigma(t)$ for $\eta_0 = 1.6 > \sqrt{2}$ [and hence $V(\eta_0) > V(0)$, see eq.(4.1)] and $g = 10^{-3}$. We find the typical behaviour of unbroken symmetry. Notice again that the effective or tree level potential is an irrelevant quantity for the dynamics, the asymptotic amplitude of oscillation of the zero mode is $\eta \approx 0.5$, which is smaller than the minimum of the tree level potential $\eta = 1$ but the oscillations are symmetric around $\eta = 0$.

Since the dynamical evolution sampled both vacua symmetrically from the beginning, there never was a symmetry breaking in the first place, and "symmetry restoration" is just the statement that the initial state has enough energy such that the *dynamics* probes both vacua symmetrically despite the fact that the tree level potential allows for broken symmetry ground states.

In their comment (hep-ph/9608341), KLS seem to agree with our conclusion that in the situations studied in our articles [6,7], when the expectation value is

released below the potential hill, symmetry restoration does not occur.

We will present a deeper analytical and numerical study of the subtle aspects of the dynamics in this case in a forthcoming article [9].

V. LINEAR VS. NONLINEAR DISSIPATION (THROUGH PARTICLE CREATION)

As already stressed, the field theory dynamics is unavoidable nonlinear for processes like preheating and reheating. It is however interesting to study such processes in the amplitude expansion. This is done in detail in refs. [6,7]. To dominant order, the amplitude expansion means to linearize the zero mode evolution equations. This approach permits an analytic resolution of the evolution in closed form by Laplace transform. Explicit integral representations for $\eta(t)$ follow as functions of the initial data [6,7]. Moreover, the results can be clearly described in terms of S-matrix concepts (particle poles, production thresholds, resonances, etc.).

Let us consider the simplest model where the inflaton Φ couples to a another scalar σ and to a fermion field ψ, and potential [7]

$$V = \frac{1}{2}\left[m_\Phi^2\Phi^2 + m_\sigma^2\sigma^2 + g\,\sigma^2\Phi^2\right] + \frac{\lambda_\Phi}{4!}\,\Phi^4 + \frac{\lambda_\sigma}{4!}\,\sigma^4 + \bar{\psi}(m_\psi + y\Phi)\psi\ .$$

In the unbroken symmetry case ($m_\Phi^2 > 0$) the inflaton is always stable and we found for the order parameter (expectation value of Φ) evolution in the amplitude expansion [7],

$$\eta(t) = \frac{\eta_i}{1-\frac{\partial\Sigma(im_\Phi)}{\partial m_\Phi^2}}\cos[m_\Phi t] + \frac{2\,\eta_i}{\pi}\int_{m_\Phi+2m_\sigma}^{\infty}\frac{\omega\Sigma_I(\omega)\cos\omega t\; d\omega}{[\omega^2-m_\Phi^2-\Sigma_R(\omega)]^2+\Sigma_I(\omega)^2}\ . \quad (5.1)$$

where $\Sigma_{\text{physical}}(i\omega\pm 0^+) = \Sigma_R(\omega)\pm i\Sigma_I(\omega)$ is the inflaton self-energy in the physical sheet, $\eta_i = \eta(0)$ and $\dot\eta(0) = 0$. The first term is the contribution of the one-particle pole (at the physical inflaton mass). This terms oscillates forever with constant amplitude. The second term is the cut contribution $\eta(t)_{cut}$ corresponding to $\Phi\to\Phi+2\sigma$.

In general, when

$$\Sigma_I(\omega\to\omega_{\text{threshold}}) \overset{\omega\to\omega_{\text{threshold}}}{=} B\;(\omega-\omega_{\text{threshold}})^\alpha\ ,$$

the the cut contribution behaves for late times as

$$\eta(t)_{cut} \simeq \frac{2\,\eta_i}{\pi}\frac{B\;\omega_{\text{threshold}}\;\Gamma(1+\alpha)}{[\omega^2-m_\Phi^2-\Sigma_R(\omega_{\text{threshold}})]^2}\; t^{-1-\alpha}\cos\left[\omega_{\text{threshold}}t + \frac{\pi}{2}(1+\alpha)\right]\ . \quad (5.2)$$

Here, $\omega_{\text{threshold}} = m_\Phi + 2M_\sigma$ and $\alpha = 2$ since to two-loops, [7]

$$\Sigma_I(\omega) \overset{\omega\to m_\Phi+2M_\sigma}{=} \frac{2g^2\pi^2}{(4\pi)^4}\frac{M_\sigma\sqrt{m_\Phi}}{(m_\Phi+2M_\sigma)^{7/2}}[\omega^2-(m_\Phi+2M_\sigma)^2]^2\ .$$

In the broken symmetry case ($m_\Phi^2 < 0$) we may have either $M < 2m_\sigma$ or $M > m_\sigma$, where M is the physical inflaton mass. ($M = |m_\Phi|\sqrt{2}$ at the tree level). In the first case the inflaton is stable and eq.(5.1) holds. However, the self-energy starts now at one-loop and vanishes at threshold with a power $\alpha = 1/2$. For $M > m_\sigma$ the inflaton decomes a resonance (an unstable particle) with width (inverse lifetime)

$$\Gamma = \frac{g^2\Phi_0^2}{8\pi M}\sqrt{1-\frac{4m_\sigma^2}{M^2}}\ .$$

This pole dominates $\eta(t)$ for non asymptotic times

$$\delta(t) \simeq \delta_i\,A\,e^{-\Gamma t/2}\;\cos(Mt+\gamma)\ , \quad (5.3)$$

where

$$A = 1 + \frac{\partial\Sigma_R(M)}{\partial M^2}\ , \qquad \gamma = -\frac{\partial\Sigma_I(M)}{\partial M^2}\ . \quad (5.4)$$

In summary, eq.(5.3) holds provided: a) the inflaton is a resonance and b) $t \le \Gamma^{-1}\ln(\Gamma/M_\sigma)$. For later times the fall off is with a power law $t^{-3/2}$ determined by the spectral density at threshold as before [7].

In ref. [7] the selfconsistent nonlinear evolution is computed to one-loop level for the model (5.1). In fig. 8 $\eta(t)$ is plotted as a function of time for $\lambda = g = 1.6\pi^2,\ y = 0,\ m_\sigma = 0.2m_\Phi,\ \eta(0) = 1$ and $\dot\eta(0) = 0$.

Figure 8 shows a very rapid, non-exponential damping within few oscillations of the expectation value and a saturation effect when the amplitude of the oscillation is rather small (about 0.1 in this case), the amplitude remains almost constant at the latest times tested. Figures 8 and 9 clearly show that the time scale for dissipation (from fig. 8) is that for which the particle production mechanism is more efficient (fig. 9). Notice that the total number of particles produced rises on the same time scale as that of damping in fig. 8 and eventually when the expectation value oscillates with (almost) constant amplitude the average number of particles produced remains constant. This behaviour is a close analog to the selfcoupled inflaton for unbroken symmetry (fig.1). The amplitude expansion predictions are in qualitative agreement with both results.

These figures clearly show that damping is a consequence of particle production. At times larger than about $40\ m_\Phi^{-1}$ (for the initial values and couplings chosen) there is no appreciable damping. The amplitude is rather small and particle production has practically shut off. If we had used the *classical* evolution of the expectation value in the mode equations, particle production would not shut off (parametric resonant amplification), and thus we clearly

see the dramatic effects of the inclusion of the back reaction.

In ref. [7] the broken symmetry case $m_\Phi^2 < 0$ is then studied. Figures 11-13 show $\eta(\tau)$ vs τ, $\mathcal{N}_\sigma(\tau)$ vs τ and $\mathcal{N}_{q,\sigma}(\tau = 200)$ vs q respectively, for $\lambda/8\pi^2 = 0.2$; $g/\lambda = 0.05$; $m_\sigma = 0.2\,|m_\Phi|$; $\eta(0) = 0.6$; $\dot{\eta}(0) = 0$. Notice that the mass for the linearized perturbations of the Φ field at the broken symmetry ground state is $\sqrt{2}\,|m_\Phi| > 2m_\sigma$. Therefore, for the values used in the numerical analysis, the two-particle decay channel is open. For these values of the parameters, linear relaxation predicts exponential decay with a time scale $\tau_{rel} \approx 300$ (in the units used). Figure 11 shows very rapid non-exponential damping on time scales about *six times shorter* than that predicted by linear relaxation. The expectation value reaches very rapidly a small amplitude regime, once this happens its amplitude relaxes very slowly. In the non-linear regime relaxation is clearly *not* exponential but extremely fast. The amplitude at long times seems to relax to the expected value, shifted slightly from the minimum of the tree level potential at $\eta = 1$. This is as expected from the fact that there are quantum corrections. Figure 12 shows that particle production occurs during the time scale for which dissipation is most effective, giving direct proof that dissipation is a consequence of particle production. Asymptotically, when the amplitude of the expectation value is small, particle production shuts off. We point out again that this is a consequence of the back-reaction in the evolution equations. Without this back-reaction, as argued above, particle production would continue without indefinitely. Figure 13 shows that the distribution of produced particles is very far from thermal and concentrated at low momentum modes $k \leq |m_\Phi|$. This distribution is qualitatively similar to that in the unbroken symmetry case, and points out that the excited state obtained asymptotically is far from thermal.

In ref. [7] the case where the inflaton is only coupled to fermions is studied ($g = 0,\ y \neq 0$). The damping of the zero mode is very inefficient in such case due to Pauli blocking. Namely, the Pauli exclusion principle forbids the creation of more than 2 fermions per momentum state. Pauli blocking shuts off particle production and dissipation very early on.

VI. FUTURE PERSPECTIVES

The preheating and reheating theory in inflationary cosmology is currently a very active area of research in fast development, with the potential for dramatically modifying the picture of the late stages of inflationary phase transitions.

As remarked before, estimates and field theory calculations have been done mostly assuming Minkowski spacetime. Results in de Sitter [10] and FRW backgrounds [28] are just beginning to emerge. A further important step will be to consider the background dynamics. Namely, the coupled gravitational and matter field dynamics. The matter state equations obtained in Minkowski [8] and de Sitter backgrounds [9] give an indication, through the Einstein-Friedmann equation, of the scale factor behaviour.

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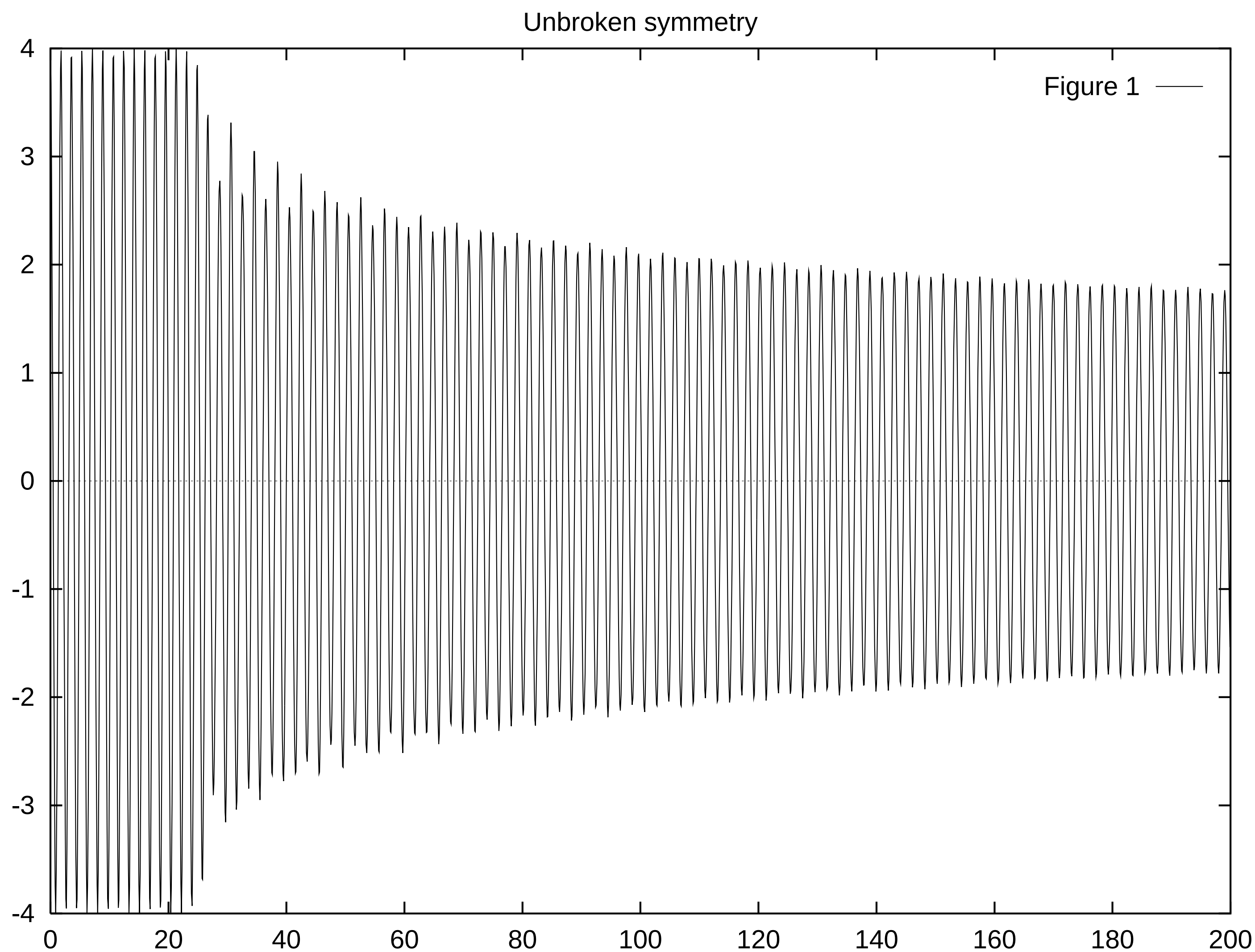

Figure 1: $\eta(\tau)$ vs. τ for the unbroken symmetry case with $\eta_0 = 4$, $g = 10^{-12}$.

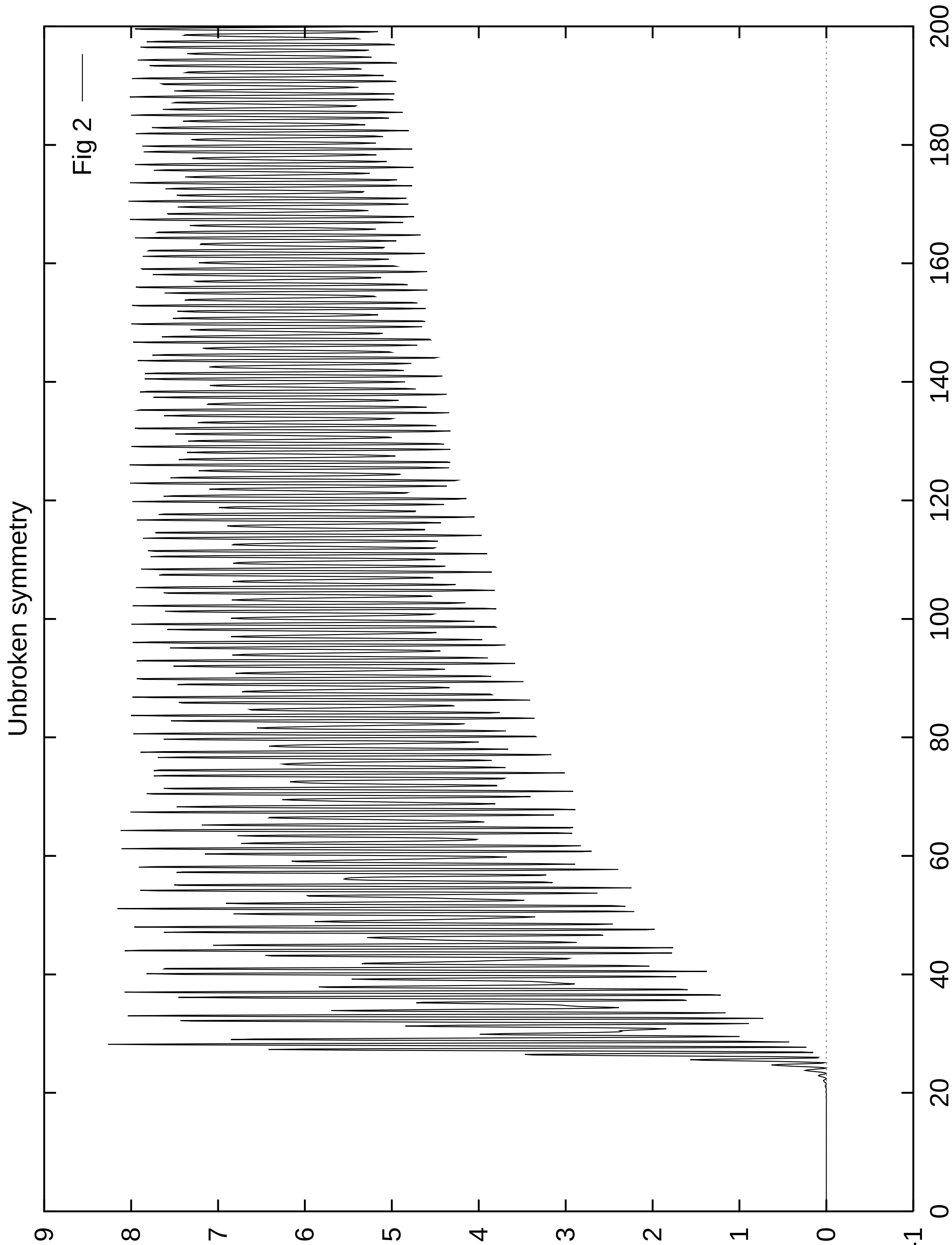

Figure 2: $g\Sigma(\tau)$ vs. τ for the same values of the parameters as in Fig. 1.

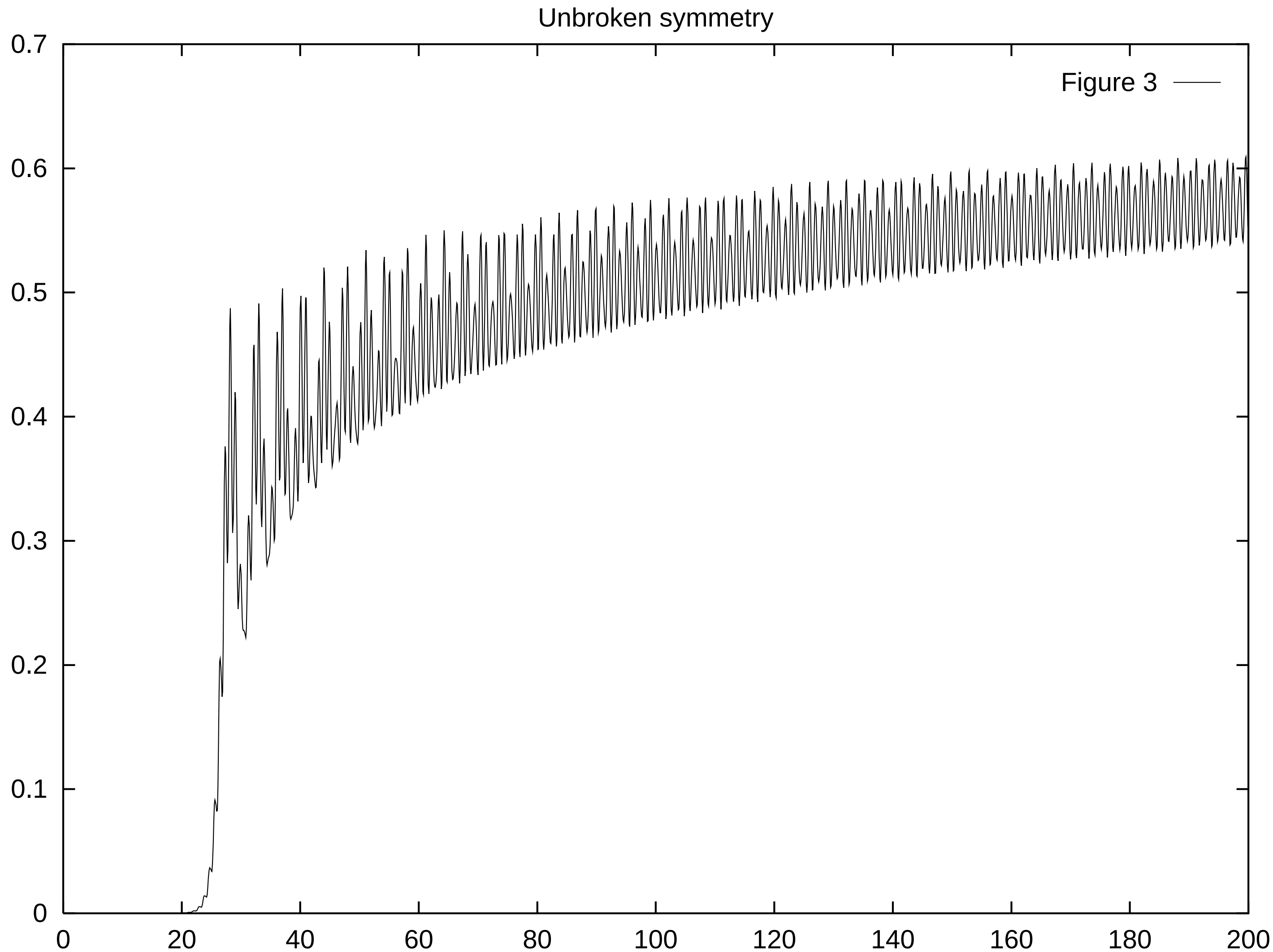

Figure 3: $g\mathcal{N}(\tau)$ for the same parameters as in fig. 1.

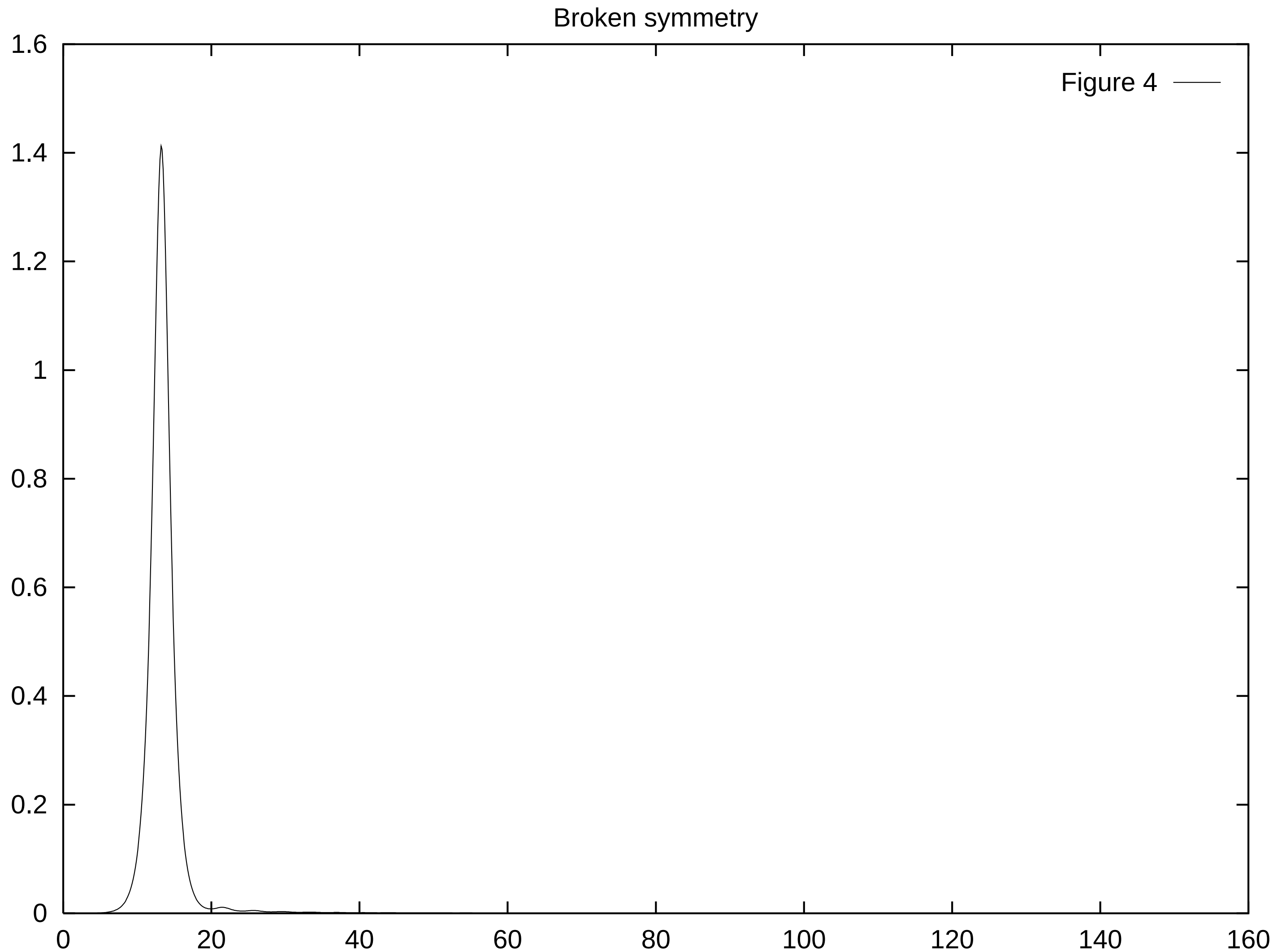

Figure 4: $\eta(\tau)$ vs. τ for the broken symmetry case with $\eta_0 = 10^{-5}$, $g = 10^{-12}$.

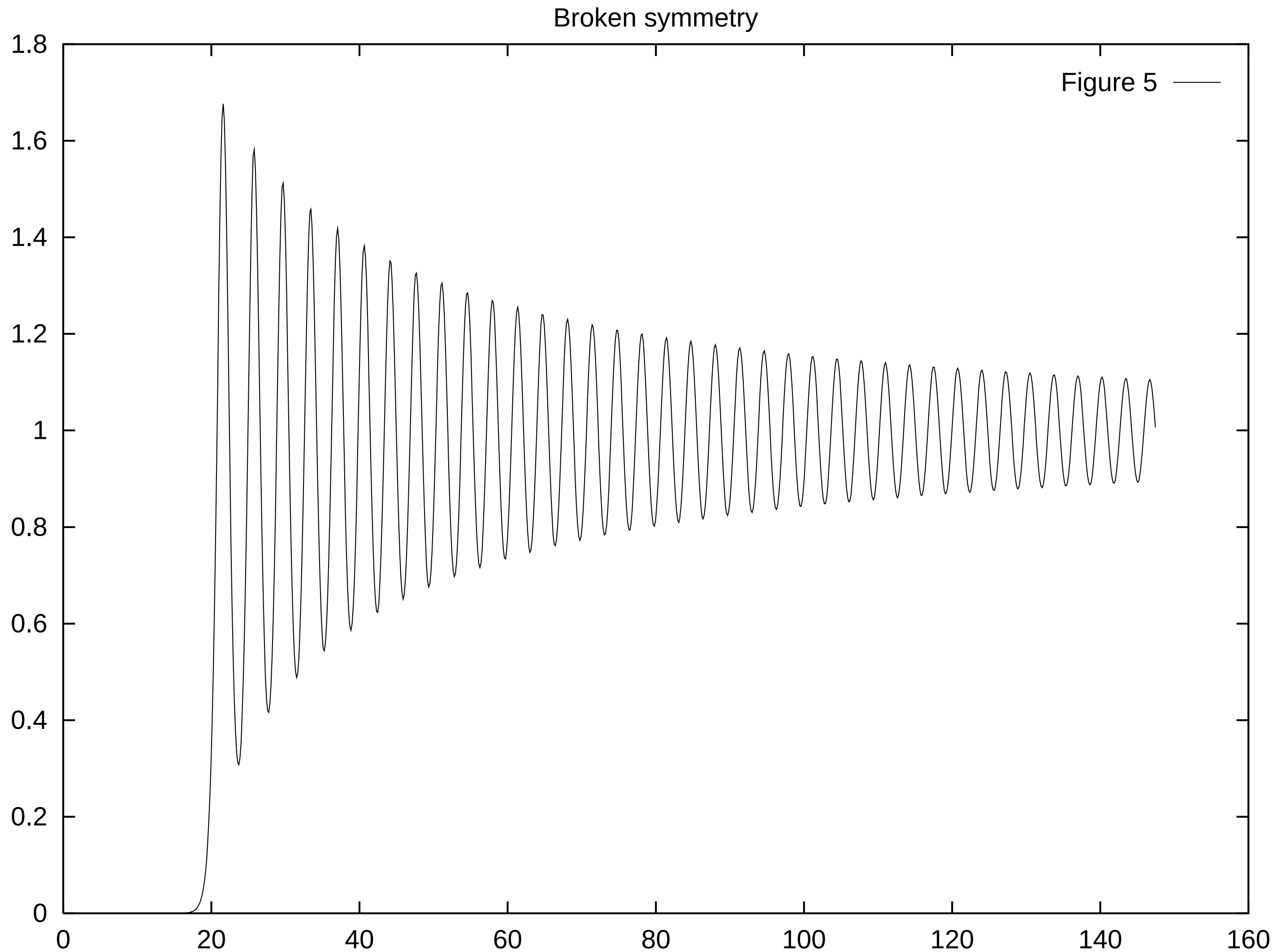

Figure 5: $g\Sigma(\tau)$ for the same values of the parameters as in fig. 4.

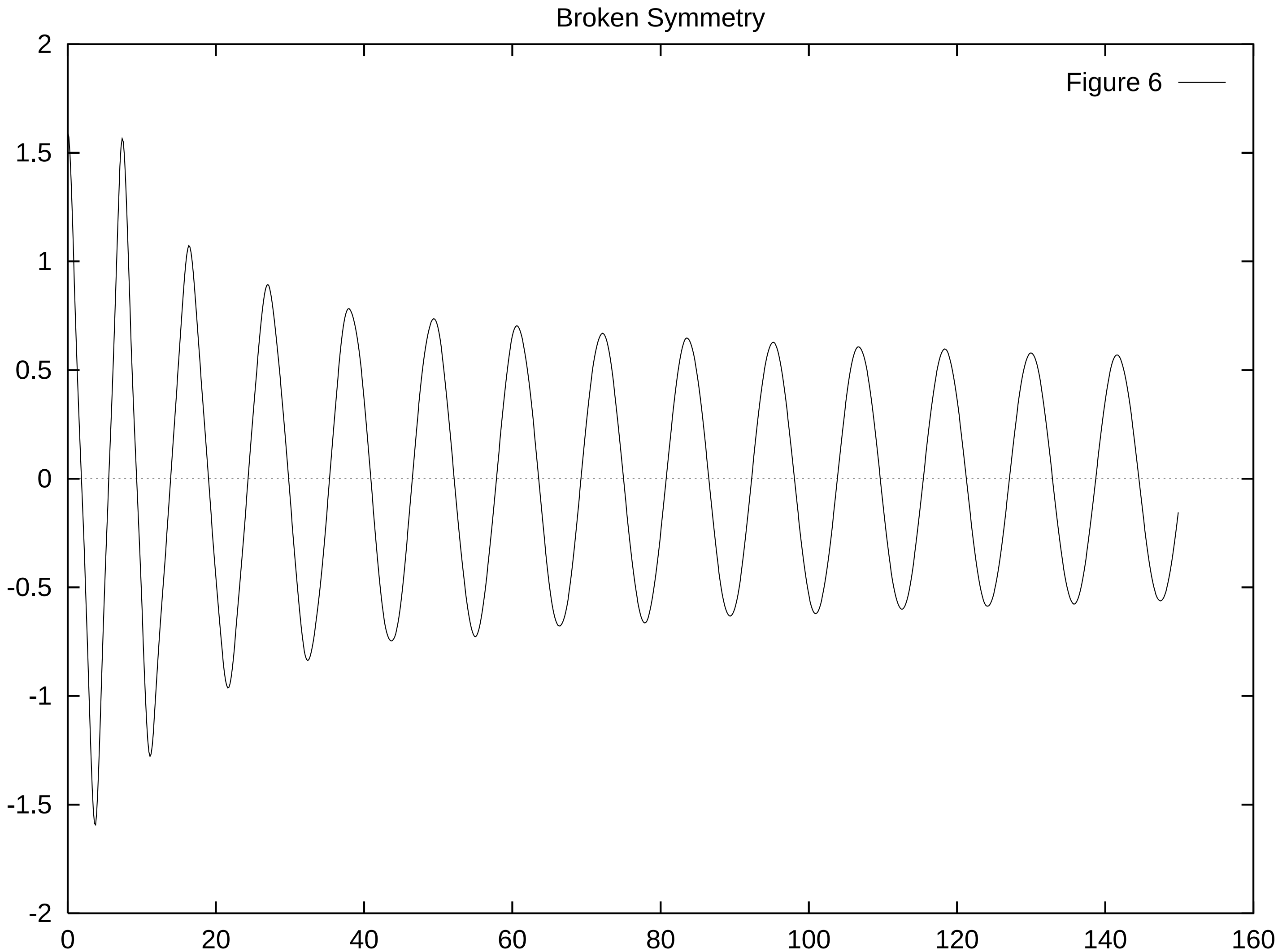

Figure 6: $\eta(\tau)$ vs. τ for the broken symmetry case with $\eta_0 = 1.6 > \sqrt{2}$, $g = 10^{-3}$.

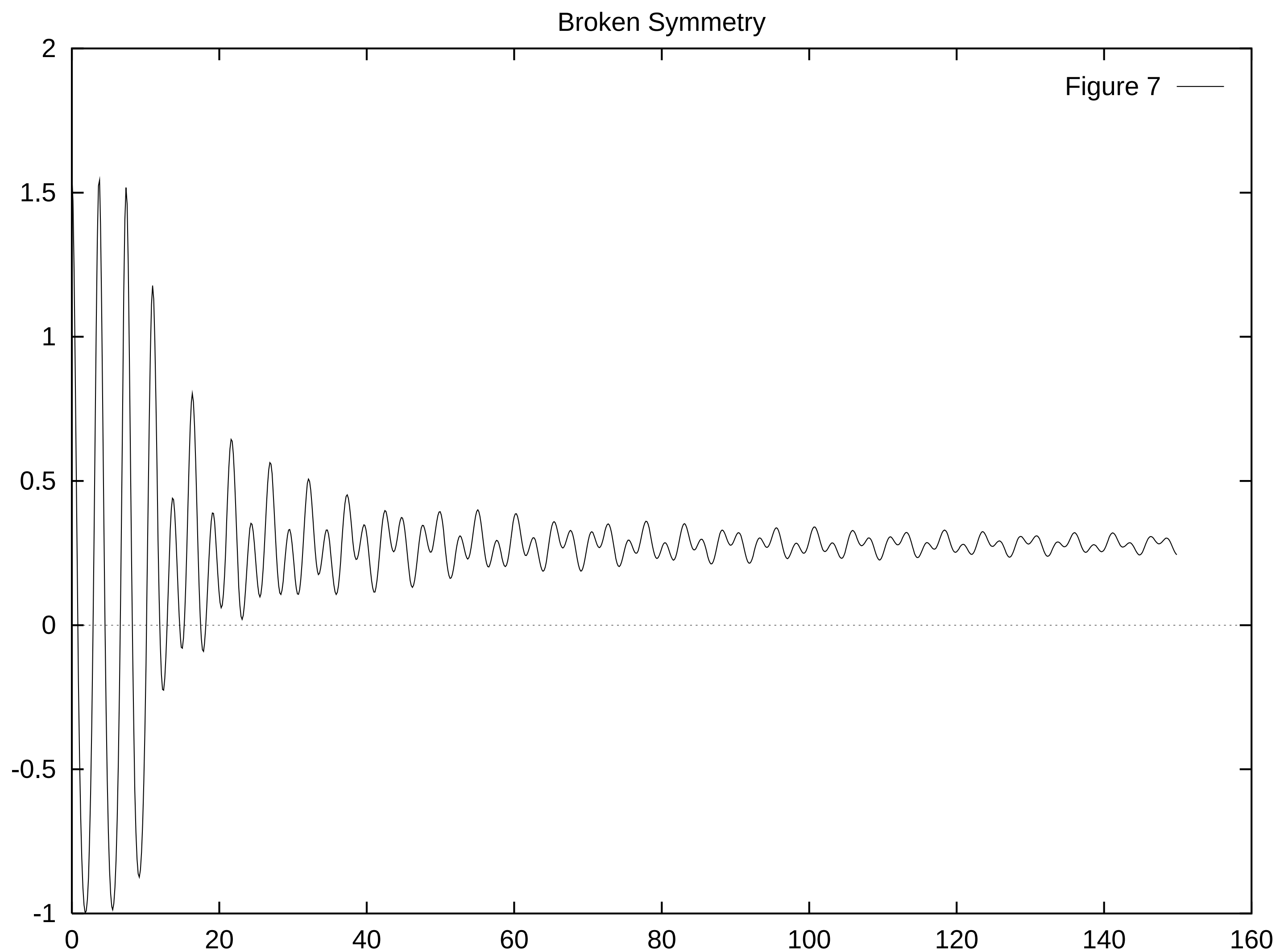

Figure 7: The effective mass squared vs. τ for the same values of the parameters as in fig. 6.

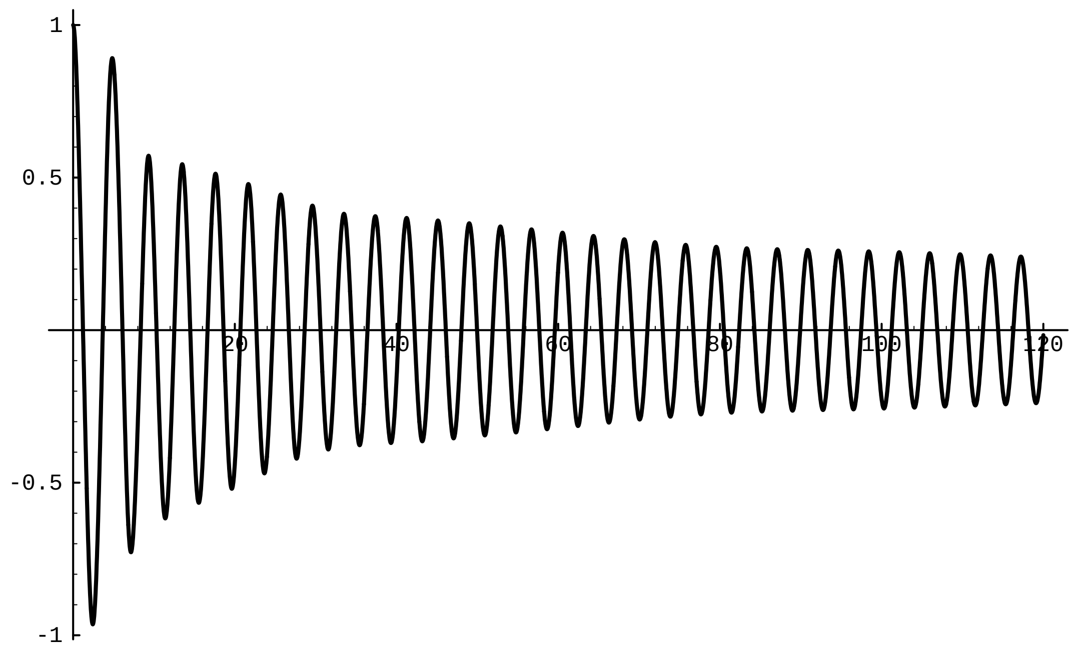

Figure 8: The inflaton coupled to a lighter scalar field σ: $\eta(\tau)$ vs τ for the values of the parameters $y = 0$; $\lambda/8\pi^2 = 0.2$; $g = \lambda$; $m_\sigma = 0.2\, m_\phi$; $\eta(0) = 1.0$; $\dot{\eta}(0) = 0$.

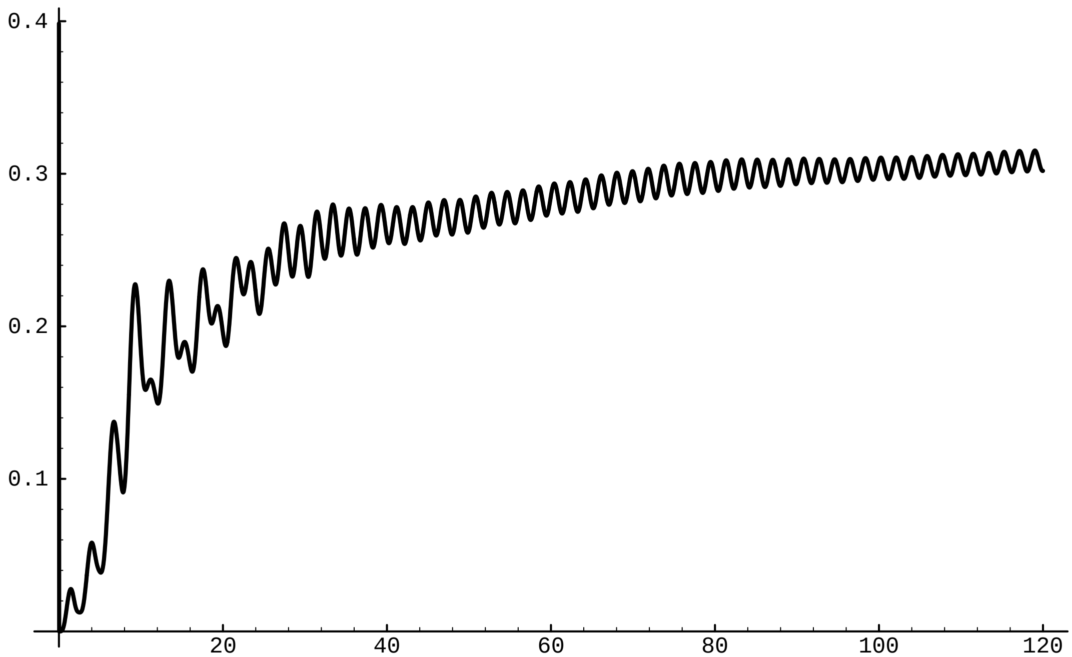

Figure 9: $\mathcal{N}_\sigma(\tau)$ vs. τ for the same value of the parameters as figure 8.

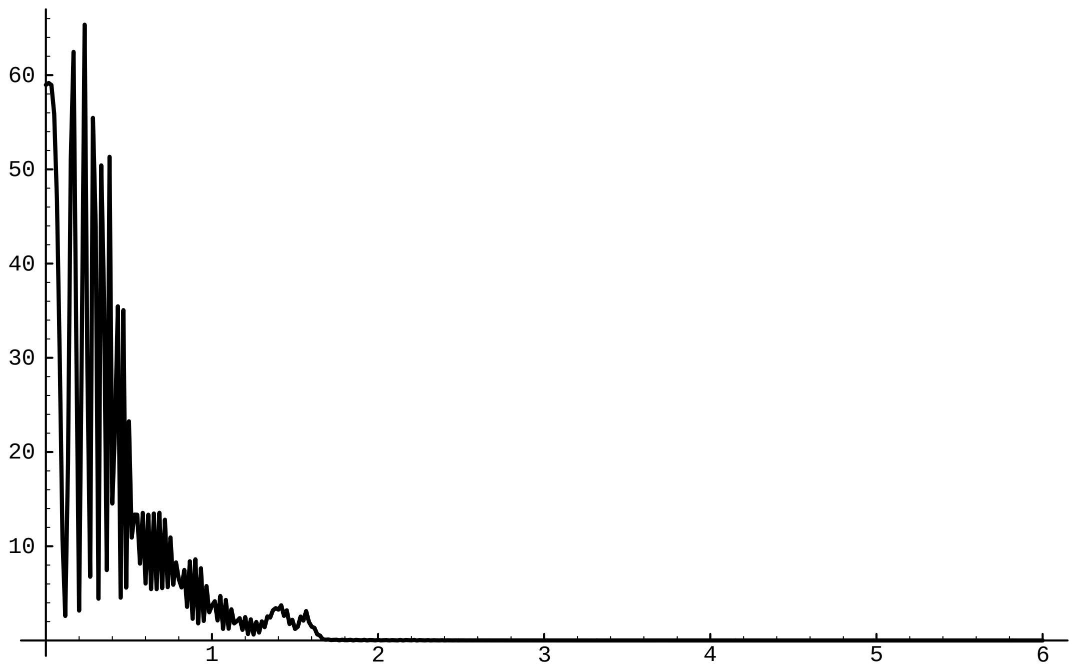

Figure 10: $\mathcal{N}_{q,\sigma}(\tau = 120)$ vs. q for the same values as in fig. 8.

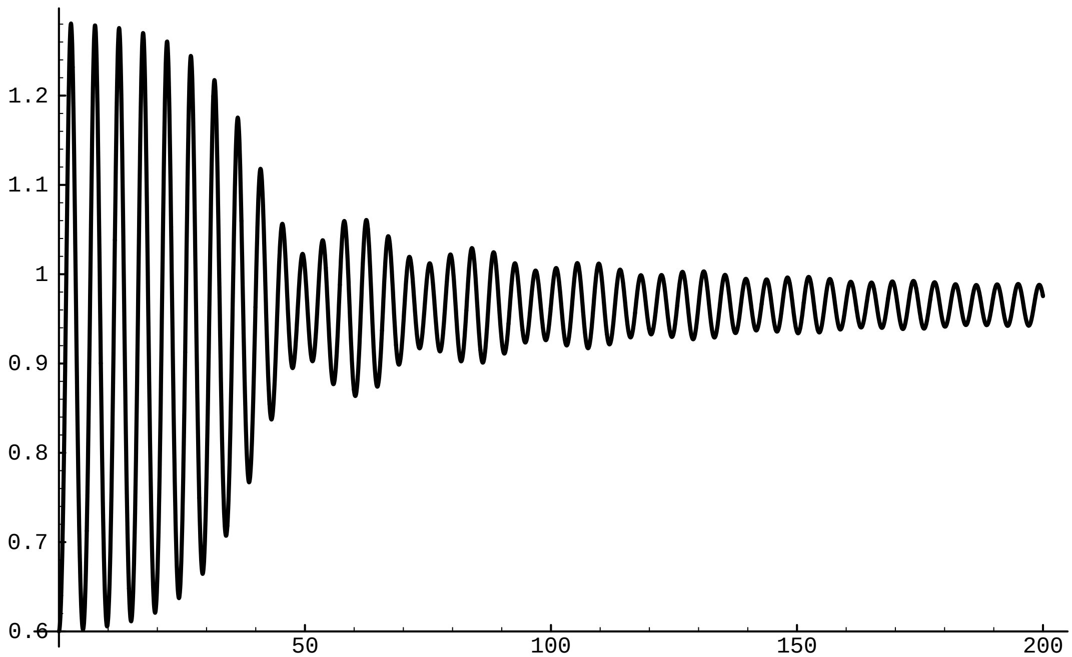

Figure 11: The inflaton in the broken symmetry case coupled to a lighter scalar σ. $\eta(\tau)$ vs τ for the values of the parameters $y = 0$; $\lambda/8\pi^2 = 0.2$; $g = \lambda$; $m_\sigma = 0.2\,|m_\phi|$; $\eta(0) = 0.6$; $\dot{\eta}(0) = 0$.

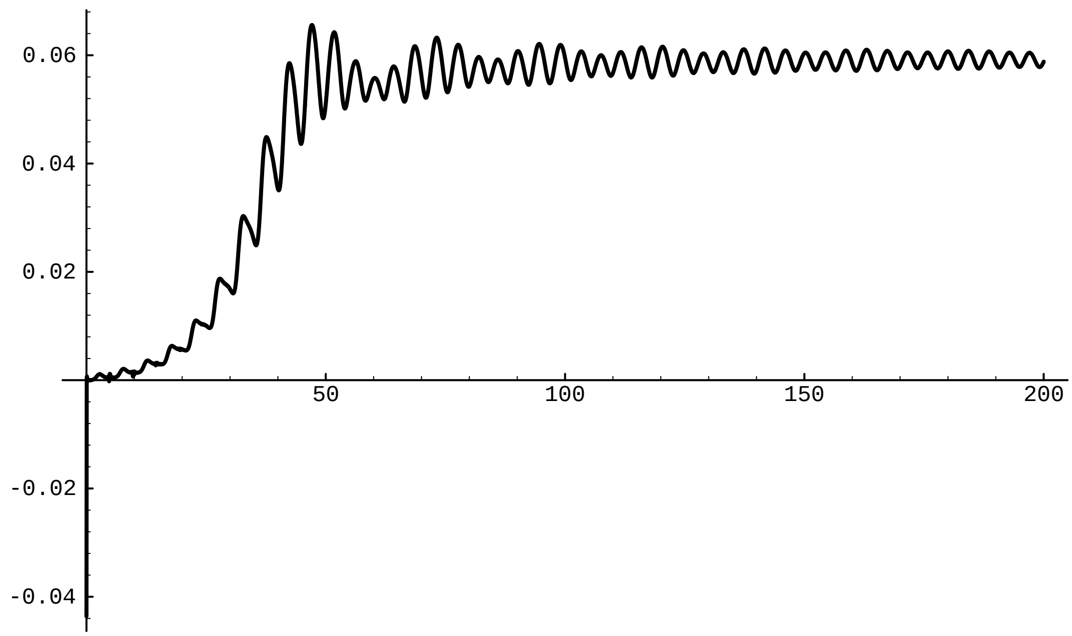

Figure 12: $\mathcal{N}_\sigma(\tau)$ vs. τ for the same value of the parameters as fig. 11.

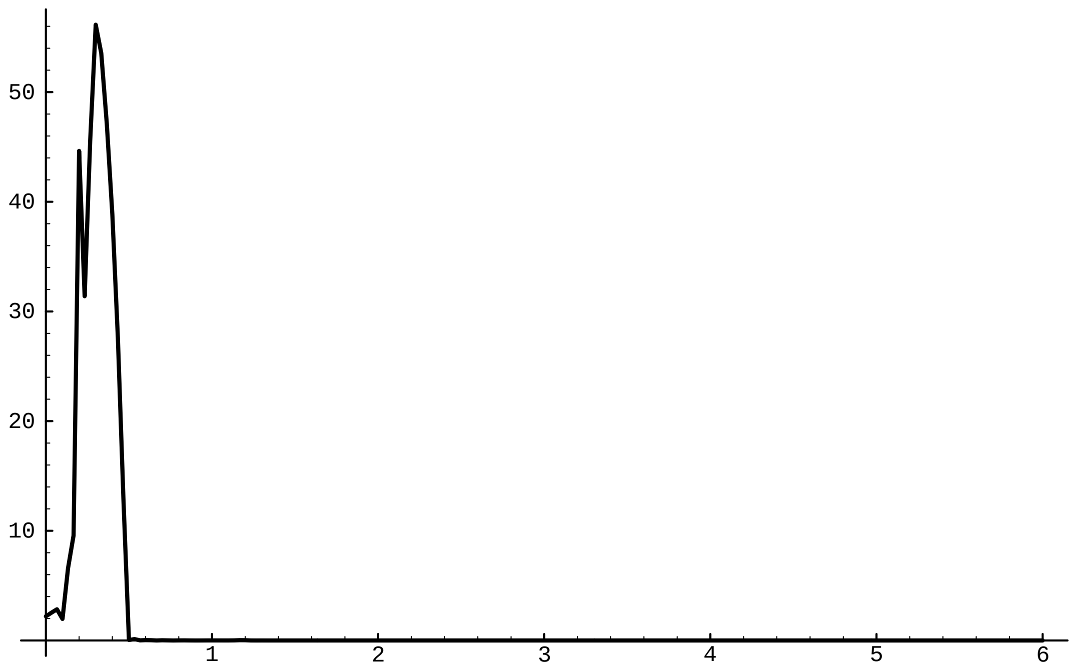

Figure 13: $\mathcal{N}_{q,\sigma}(\tau = 200)$ vs. q for the same values as in fig. 11.